HEAT AND MASS EXCHANGE IN THE COOLING OF AIR WHICH IS IN DIRECT CONTACT WITH A COOLING LIQUID

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We give a theoretical solution which can be used for determining the average temperature of humid saturated air.

In a number of cases it becomes necessary to cool humid saturated air which is at temperature $t \leqslant 0°C$ and is in direct contact with a cooling liquid. The energy equation for this process can be written as follows:

$$\frac{\partial t}{\partial \tau} + w_x \frac{\partial t}{\partial x} + w_y \frac{\partial t}{\partial y} + w_z \frac{\partial t}{\partial z} = a\nabla^2 t + \frac{D}{\rho c_p} \nabla^2 (mi_a). \tag{1}$$

In deriving Eq. (1) it was assumed that the work done by external forces and the kinetic energy of the stream are negligibly small in comparison with its enthalpy; there are no internal sources of heat, and the thermophysical characteristics of the air are constant.

As is known, a system of differential equations describing convective heat exchange when the two media are in direct contact includes not only the energy equation but also the equations of heat exchange, motion, and continuity. To this system we must add conditions for mechanical and thermal interaction at the interface between the media and initial conditions. Solving such a system of equations in general form is a rather difficult problem, and no solution has yet been found. The equation of the temperature field can be obtained by using other methods of solution. One of them is described below. We consider the problem of the cooling of air which moves in a channel; with medium cooling the air moves along the walls of the channel in the same direction and is in direct contact with it.

Figure 1 shows a diagram of this process.

Air in a supersaturated state, usually referred to as ice fog, passes through the first cross section (I). The enthalpy I_1 of this air can be represented as follows:

$$I_1 = M_a i_a + M_v i_v + M_i i_i. \tag{2}$$

Through the second cross section (II) passes an ice fog whose enthalpy is given by the expression

$$I_2 = M_a (i_a - di_a) + (M_v - dM_i)(i_v - di_v) + (M_i + dM_i^I)(i_i - di_i). \tag{3}$$

For the elementary volume of air under consideration, enclosed between the first and second cross sections, which are separated by a distance dx, we can write

$$dI = dQ_1. \tag{4}$$

But if there is a temperature difference between the surface of the liquid and the air moving in the channel, there will be heat exchange. On the surface of the liquid itself the partial pressure of the saturated water vapor, which is in thermodynamic equilibrium with the liquid, will be lower than the partial pressure at the center of the stream. The difference between the concentrations of the water vapor at different points of the moving stream of air causes mass exchange. The elementary surface of the cooling liquid enclosed between cross sections I and II will absorb an amount of heat

$$dQ_2 = \alpha\vartheta dF + dM_i^{II} i_i, \tag{5}$$

where $dM_i^{II} = (1 - k)dM_i$ is the amount of ice formed in the elementary volume of air (enclosed between cross sections I and II) and precipitated on the heat-exchange surface.

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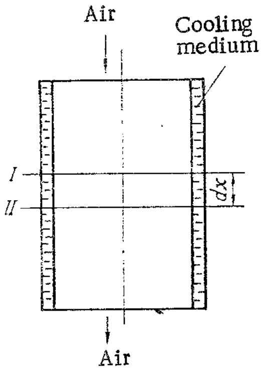

Fig. 1. Diagram of the apparatus.

In formulating Eq. (5), we assumed that the second term on the right side takes account of the mass exchange. In accordance with the law of conservation of energy,

$$|dQ_1| = |dQ_2| \tag{6}$$

or

$$dQ_1 = -dQ_2. \tag{7}$$

In Eq. (7) the minus sign is used because the heat is being removed from the air. If we consider Eqs. (2)-(5) and Eq. (7) together, we obtain

$$\alpha\vartheta dF + (1-k)\,dM_i\,i_i = i_i k dM_i - M_a di_a - M_v di_v - i_v dM_i. \tag{8}$$

In Eq. (8) we have disregarded second-order terms. In our further discussion of this equation, we take account of the following:

$$M_v = d''M_a, \tag{9}$$

$$M_i = d_i M_a. \tag{10}$$

In the region of negative temperatures (from 0°C to −30°C)

$$i_v = At + B, \tag{11}$$

$$i_i = A_1 t + B_1, \tag{12}$$

$$d'' = at^2 + bt + c, \tag{13}$$

$$d_i = a_1 t^2 + b_1 t, \tag{14}$$

where A = 1.75 kJ/(kg•°K); B = 2501 kJ/kg; A_1 = −2.1 kJ/(kg•°K); B_1 = 335.2 kJ/kg; a = 5.25•10^{-6} 1/°K^2; b = 2.58•10^{-4} 1/°K; c = 3.83•10^{-3}; a_1 = −5.25•10^{-6} 1/°K^2; b_1 = 2.58•10^{-4} 1/°K.

Formulas (11) and (12) are valid for temperatures $0 \geqslant t \geqslant -30$°C, while (13) and (14) are valid for $0 \geqslant t \geqslant -20$°C.

Taking account of Eqs. (9)-(14), we can write Eq. (8) as follows:

$$\alpha dF = -\left(A_2\vartheta + A_3 + \frac{A_4}{\vartheta}\right) d\vartheta, \tag{15}$$

where

$$A_2 = M_a\,[3a_1A_1 + A\,(a+2a_1) - 4a_1Ak] = M_a(2.39 - 4.41k)\cdot 10^{-5},\ \text{kW/°K}^3;$$

$$A_3 = M_a\,\{t_{su}\,[6a_1A_1 + A\,(a+2a_1) - 8a_1A_1k] + 2a_1\,(B+B_1) + 2b_1A_1 + A\,(b_1+b) - 2k\,(2a_1B_1 + b_1A_1)\},$$

or

$$A_3 = M_a\,[t_{su}(4.78 - 8.82k)\cdot 10^{-5} - 2.17\cdot 10^{-2} + 5.96\cdot k\cdot 10^{-3}],\ \text{kW/°K}^2;$$

$$A_4 = M_a\,\{t_{su}^2\,[3a_1A_1 + A\,(a+2a_1) - 4a_1A_1k] + t_{su}\,[2a_1\,(B_1+B) + A\,(b+b_1) + 2b_1A_1 - 2k\,(2a_1B_1 + b_1A_1)] + b_1B_1 + 2b_1B_1k + c_p + cA - b_1B\},$$

TABLE 1. Comparison of Theoretical and Experimental Values of Temperatures, °C

t_{th}	t_{exp}	Discrepancy, %
—7,72	—7,80	+1,03
—5,41	—5,31	—2,03
—7,90	—8,00	+1,25
—19,69	—20,20	+2,52
—20,10	—19,53	—2,84
—20,53	—20,00	—2,58
—20,50	—20,00	—2,44
—17,45	—17,60	+0,85
—16,53	—16,20	—2,00

or

$$A_4 = M_a\,[t_{su}^2 (2.39 - 4.41k)\cdot 10^{-5} + t_{su}\,(k\cdot 5.96\cdot 10^{-3} - 2.87\cdot 10^{-2}) + 0.277 + 0.865k],\ \text{kW/°K}.$$

Integrating Eq. (15), we obtain

$$\alpha F = \frac{1}{2} A_2 (\vartheta_0^2 - \vartheta^2) + A_3 (\vartheta_0 - \vartheta) + A_4 \ln \frac{\vartheta_0}{\vartheta}\ . \qquad (16)$$

Here we have assumed that α = const; t_{su} = const; k = const; c_p = const.

Equation (16) enables us to determine the average temperature at any cross section of the apparatus.

A comparison of the theoretical results with the experimental data is shown in Table 1. As can be seen, a comparison of the theoretical and experimental data shows satisfactory agreement. The discrepancy does not exceed 3%.

NOTATION

D, coefficient of molecular diffusion; t, temperature; τ, time; w_x, w_y, w_z, components of velocity; a, thermal diffusivity; ρ, density; c_p, heat capacity; m, mass concentration; M_a, M_v, M_i, masses of dry air, dry saturated water vapor, and ice; i_a, i_v, i_i, enthalpy of dry air, water vapor, and ice; dQ_1, heat released by cooled air; k, fraction of ice formed in the elementary volume; d", moisture content of humid saturated air; d_i, amount of ice in 1 kg of dry air; α, heat-transfer coefficient; $\vartheta = t - t_{su}$, temperature difference between the air and the surface of the liquid; ϑ_0, temperature difference at the inlet of the apparatus; F, heat-exchange surface; t_{th}, t_{exp}, theoretical and experimental values of temperature.